hw set 23
sect 8.4
p 593

3. $\frac{4}{7} - \frac{4}{8} + \frac{4}{9} - \frac{4}{10} + \frac{4}{11} - \ldots$

$$= \sum_{n=7}^{\infty} \frac{4}{n} (-1)^{n+1}$$

Alternating Series test

$$\lim_{n\to\infty} \frac{4}{n} = 0$$

$\therefore$ converges

4. $-\frac{1}{3} + \frac{2}{4} - \frac{3}{5} + \frac{4}{6} - \frac{5}{7} + \ldots$

$$\sum_{n=1}^{\infty} (-1)^n \frac{n}{(n+2)}$$

Alternating Series test

$$\lim_{n\to\infty} \frac{n}{n+2} = 1 \neq 0$$

$\therefore$ diverges

19. $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^n}{n^3}$

consider $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n^3}$

$$l = \lim_{n\to\infty} \left| \frac{\frac{3^{n+1}}{(n+1)^3}}{\frac{3^n}{n^3}} \right| = \lim_{n\to\infty} \left| \frac{3^{n+1}}{(n+1)^3} \cdot \frac{n^3}{3^n} \right|$$

$$= \lim_{n\to\infty} \frac{3n^3}{(n+1)^3} = \lim_{n\to\infty} \frac{3n^3}{n^3+3n^2+3n+1}$$

$$= 3 > 1$$

$\therefore$ diverges

$\therefore$ not absolutely convergent

20. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ all positive terms

$$l = \lim_{n\to\infty} \left(\frac{\frac{(n+1)^2}{2^{n+1}}}{\frac{n^2}{2^n}} \right)$$

$$= \lim_{n\to\infty} \frac{(n+1)^2}{2^{n+1}} \cdot \frac{2^n}{n^2}$$

$$= \lim_{n\to\infty} \frac{n^2+2n+1}{2n^2} = \frac{1}{2} < 1$$

$\therefore$ convergent

$\therefore$ absolutely convergent

21. $\displaystyle\sum_{n=0}^{\infty} \frac{(-10)^n}{n!}$

$$l = \lim_{n\to\infty} \left| \frac{\frac{(-10)^{n+1}}{(n+1)!}}{\frac{(-10)^n}{n!}} \right| = \lim_{n\to\infty} \left| \frac{(-10)^{n+1}}{(n+1)!} \cdot \frac{n!}{(-10)^n} \right| = \lim_{n\to\infty} \left| \frac{(-10)}{n+1} \right|$$

$$= \lim_{n\to\infty} \frac{10}{n+1} = 0 < 1$$

$\therefore$ converges absolutely

Homework Set 23 (part 2)

Determine the convergence of each series below.

$$\sum\left(\frac{2n+1}{n-5}\right)^n$$

$$\ell = \lim_{n\to\infty}\left[\left(\frac{2n+1}{n-5}\right)^n\right]^{1/n} = \lim_{n\to\infty}\frac{2n+1}{n-5} = 2 > 1$$

$\therefore$ diverges

$$\sum\left(\frac{-2n}{3n+1}\right)^{3n}$$

$$\ell = \lim_{n\to\infty}\left[\left|\left(\frac{-2n}{3n+1}\right)^{3n}\right|\right]^{1/n} = \lim_{n\to\infty}\left(\left|\frac{-2n}{3n+1}\right|\right)^3$$

$$= \left(\lim_{n\to\infty}\left|\frac{-2n}{3n+1}\right|\right)^3 = \left(\left|-\tfrac{2}{3}\right|\right)^3 = +\tfrac{8}{27} < 1$$

$\therefore$ converges

$$\sum\frac{e^{2n}}{n^n}$$

$$\ell = \lim_{n\to\infty}\left[\left(\frac{e^{2n}}{n^n}\right)^{1/n}\right] = \lim_{n\to\infty}\frac{e^2}{n} = 0 < 1$$

$\therefore$ converges

$$\sum\left(2\sqrt[n]{n}+1\right)^n$$

$$\ell = \lim_{n\to\infty}\left[\left(2\sqrt[n]{n}+1\right)^n\right]^{1/n} = \lim_{n\to\infty} 2n^{1/n}+1 = 2\cdot 1+1 = 3 > 1$$

$$\lim_{n\to\infty}\ln n^{1/n} = \lim_{n\to\infty}\frac{\ln n}{n} = \lim_{n\to\infty}\frac{1/n}{1} = \lim_{n\to\infty}\frac{1}{n} = 0, \text{ so } \lim_{n\to\infty} n^{1/n} = e^0 = 1$$

$\therefore$ diverges

$$\sum\frac{5^{n/2}(-2n)^{3n}}{(1+\ln(n))^n}$$

$$\ell = \lim_{n\to\infty}\left[\left|\frac{5^{n/2}(-2n)^{3n}}{(1+\ln(n))^n}\right|\right]^{1/n} = \lim_{n\to\infty}\frac{\sqrt{5}\,(2n)^3}{1+\ln(n)} = \lim_{n\to\infty}\frac{8\sqrt{5}\,n^3}{1+\ln(n)}$$

$$= \lim_{n\to\infty}\frac{24\sqrt{5}\,n^2}{1+1/n} = \lim_{n\to\infty}\frac{24\sqrt{5}\,n^3}{n+1}$$

$$= \lim_{n\to\infty}\frac{72\sqrt{5}\,n^2}{1} = \infty$$

$\therefore$ diverges